The Casimir Effect

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11 March 2009

Introduction

What is the Casimir effect?

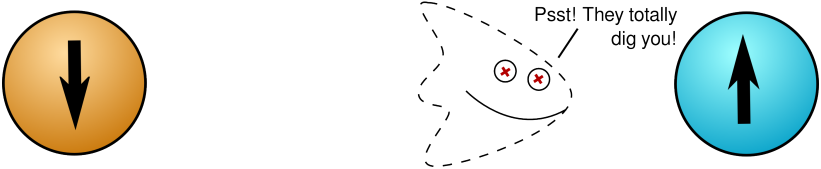
- A quantum-mechanical phenomenon
- A force due to the electromagnetic field
- Arises from differences in the vacuum energy
- Depends on boundary conditions like the geometry of conducting plates
- No real photons involved!

History

- Verwey and Overbeek: colloidal stability
- Van der Waals forces between particles suspended in liquid should go like r^{-6}
- But they fall off faster: $r^{-}7$
- Retardation effects

Colloids

- Casimir and Polder: attractive forces between two particles
- Also between particles and walls
- Two particles: E goes as R^{-3} at short distances, R^{-4} for long
- Reproduces the London interaction at short distances!
- Long distances change thanks to retardation effects



"Oh hey, by the way, we can interpret these colloid interactions as effects of the electromagnetic field vacuum"
"That's kinda neat, huh?"
"Guess we should publish?"

Electromagnetic Fields

What is a Field?

- A mathematical entity associated with every point in space
- Subject to some field dynamics: equations which constrain the values of the field
- Can carry forces between particles

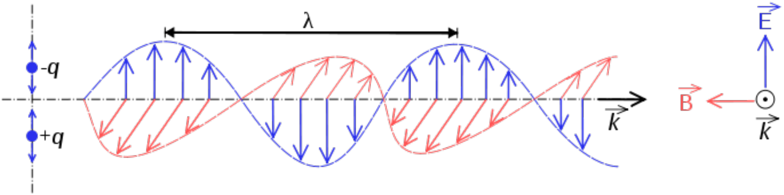
The Electromagnetic field

- The combination of two fields: the electric and magnetic field
- Carries forces between charges, like static electricity
- Carries forces between magnets, like car crushers
- Governed by Maxwell's Equations:

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\end{aligned}$$

Light as an Electromagnetic Wave

Image: wikipedia

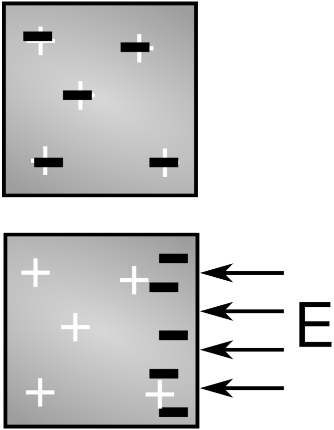


- Maxwell's equations in free space (no charges or magnets) simplify to the wave equation
- The electomagnetic field also supports oscillating electric and magnetic waves
- We identify these waves as light!

Electromagnetic Waves

- Move at c
- Have a frequency ω
- Have energy $E \propto \omega$
- Have a wavelength $\lambda \propto 1/\omega$
- Have some magnitude, or strength

Boundary Conditions



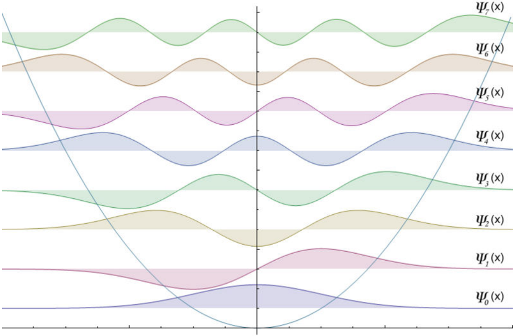
- Imposing constraints on the field changes the possible solutions
- The electric field vanishes at the surface of conductors
- Between two conductors, only some waves are possible
- These waves come in discrete frequencies

Quantum Mechanics

Quantum Harmonic Oscillators

- In quantum mechanics, things come in little chunks, or *quanta*
- Solutions to the wave equation are quantized too!
- The energy of a wave (in 1 dimension) is constrained to:

$$E_n = \hbar\omega(n + 1/2), \qquad n = 0, 1, 2, \ldots$$



The Hamiltonian

$$\hat{H} = \sum_{\mathbf{k},\lambda} \hbar\omega \left(\hat{a}^{\dagger}_{\mathbf{k},\lambda} \hat{a}_{\mathbf{k},\lambda} + \frac{1}{2} \right)$$

You literally CAN even

This is a question.

$\langle \mathit{confused} |$

We call it a "bra".

This is a state.

$| \mathit{confused} \rangle$

We call it a "ket".

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$|\mathit{confused}\rangle$

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When a bra and a ket collide, they *answer the question*

$$\begin{aligned} & \langle \mathit{confused}|\,|\mathit{confused}\rangle \\ = & \langle \mathit{confused}|\mathit{confused}\rangle \\ = & 1 \end{aligned} \tag{1}$$

The answer is the *probability* the ket satisfies the bra.

You literally CAN even

A hat over a letter means "operator".

$\hat{R}$ — The "read paper" operator

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Operators *act on states*, like functions on arguments.

$$\hat{R}\,|happy\rangle \;=\; |confused\rangle \tag{2}$$

The Hamiltonian

- The Hamiltonian operator $\hat{H}$ moves the system forward one infinitely tiny step in time
- It also defines the *energy* of any given state

The Hamiltonian

- For the electromagnetic field, the Hamiltonian is a sum (Σ)
- A bunch of operators, one for each kind of light wave
- Each possible wavelength ($\mathbf{k}$)
- Each of 2 polarizations (λ)

$$\hat{H} = \sum_{\mathbf{k},\lambda} \hbar\omega \left(\hat{a}^\dagger_{\mathbf{k},\lambda} \hat{a}_{\mathbf{k},\lambda} + \frac{1}{2} \right)$$

$\hbar$ — Quantum time-energy scale

ω — Light wave frequency

$\hat{a}^\dagger$ — Creation operator

$\hat{a}$ — Annihilation operator

Creation and annihilation operators

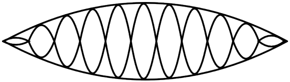
Adding photons

$$
\begin{aligned}
\hat{a}^\dagger |0\rangle &= |1\rangle \\
\hat{a}^\dagger |1\rangle &= |2\rangle \\
\hat{a}^\dagger |2\rangle &= |3\rangle
\end{aligned}
$$

Removing photons

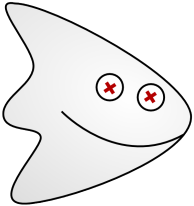
$$
\begin{aligned}
\hat{a} |2\rangle &= |1\rangle \\
\hat{a} |1\rangle &= |0\rangle \\
\hat{a} |0\rangle &= 0
\end{aligned}
$$

Photons



For physicists

$$E_n = \hbar\omega(n + 1/2), \qquad n = 0, 1, 2, \ldots$$

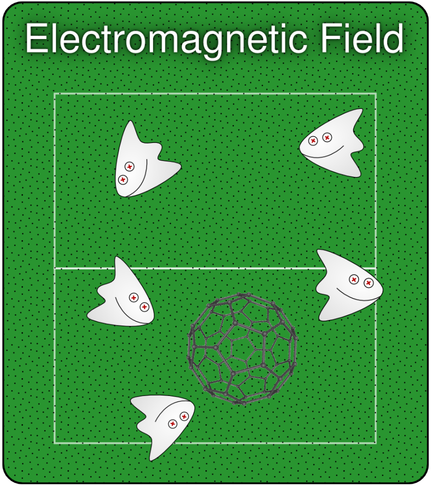


For everyone else

- $\hat{a}^\dagger$ (increase n by one) adds $\hbar\omega$ to the field energy
- That's the quantum of energy for the electromagnetic field
- We call it the "photon"

Ground State Energy

But what happens if we suck *all* the photons out of the system?



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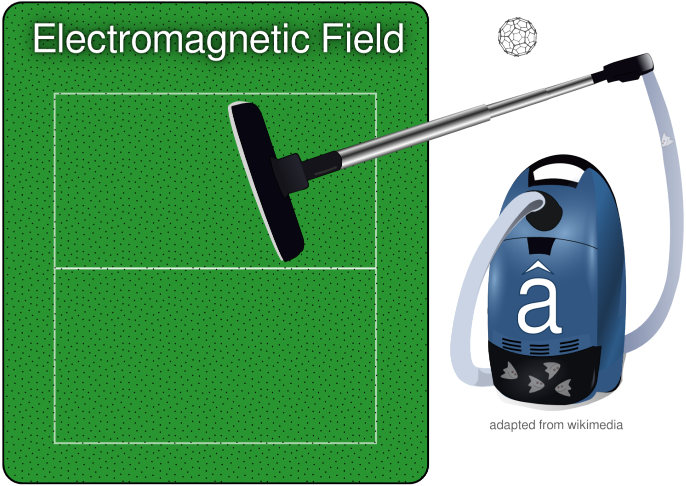


adapted from wikimedia

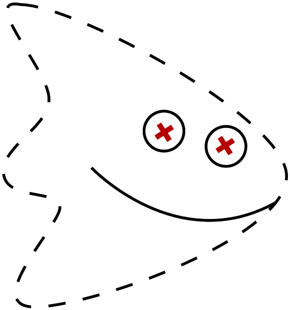
Field energy

$$
\begin{aligned}
\langle E_0 \rangle &= \langle 0| \hat{H} |0\rangle \\
\langle E_0 \rangle &= \langle 0| \sum_{\mathbf{k},\lambda} \hbar\omega \left(\hat{a}^{\dagger}_{\mathbf{k},\lambda} \hat{a}_{\mathbf{k},\lambda} + \frac{1}{2} \right) |0\rangle \\
\langle E_0 \rangle &= \langle 0| \sum_{\mathbf{k},\lambda} \hbar\omega \left(\hat{a}^{\dagger}_{\mathbf{k},\lambda} \hat{a}_{\mathbf{k},\lambda} |0\rangle + \frac{1}{2} |0\rangle \right) \\
\langle E_0 \rangle &= \langle 0| \sum_{\mathbf{k},\lambda} \hbar\omega \left(\hat{a}^{\dagger}_{\mathbf{k},\lambda} 0 + \frac{1}{2} |0\rangle \right) \\
\langle E_0 \rangle &= \langle 0| \sum_{\mathbf{k},\lambda} \hbar\omega \left(0 + \frac{1}{2} |0\rangle \right) \\
\langle E_0 \rangle &= \langle 0| \sum_{\mathbf{k},\lambda} \frac{1}{2} \hbar\omega |0\rangle
\end{aligned}
\tag{3}
$$

Field energy

$$
\begin{aligned}
\langle E_0 \rangle &= \langle 0 | \sum_{\mathbf{k},\lambda} \frac{1}{2}\hbar\omega \, | 0 \rangle \\
\langle E_0 \rangle &= \langle 0 | 0 \rangle \sum_{\mathbf{k},\lambda} \frac{1}{2}\hbar\omega \\
\langle E_0 \rangle &= \sum_{\mathbf{k},\lambda} \frac{1}{2}\hbar\omega
\end{aligned}
$$

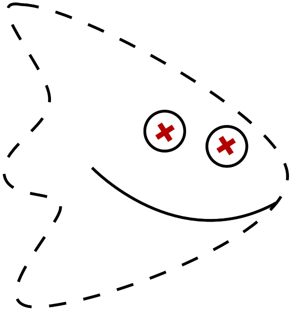
Single-frequency systems



For a single frequency **k** and polarization λ...

$$E_0 = {}^1\!/\!{}_2\hbar\omega \tag{4}$$

Single-frequency systems



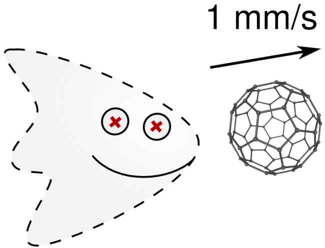
For a single frequency **k** and polarization λ...

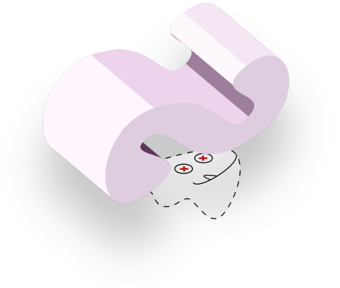
$$E_0 = {}^1\!/_2\hbar\omega \tag{4}$$

- There's *still* some energy in the field!
- This is the *vacuum*, *zero-point*, or *ground state* energy

How Much Energy?

- Green light has a wavelength of 510 nm
- That corresponds to a vacuum energy of about 1.21 eV
- Goals in their virtual soccer game happen at about 1 mm/s
- Or enough to lift a packing peanut 9.9×10^{-17} m





Infinite Energy!!!11

But the electromagnetic field (in open space) supports an infinite number of frequencies!

$$\begin{aligned} \langle E_0 \rangle &= \sum_{\mathbf{k},\lambda}^{\infty} \frac{1}{2}\hbar\omega \\ \langle E_0 \rangle &= \infty \end{aligned}$$

- The total ground state energy is the sum over the ground energy for each possible wave (mode)
- Empty space contains infinite quantities of energy
- Wait, what?

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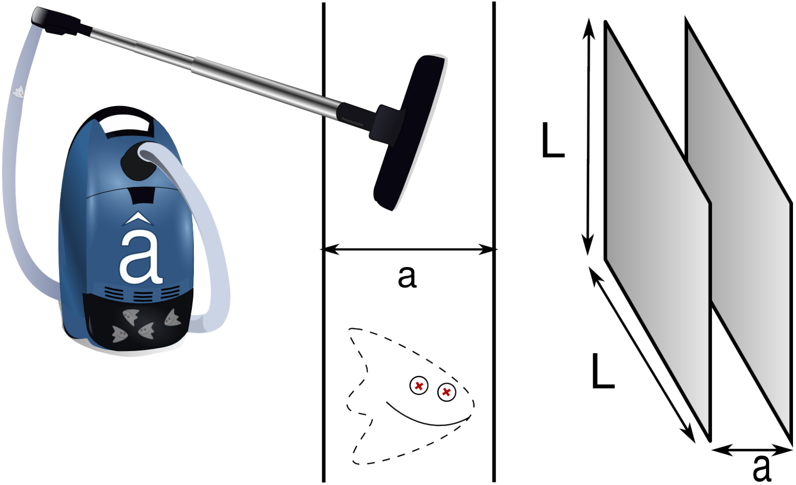
- Wait, what?
- It's only *differences* in energy that are measurable
- Where would we see an energy difference?

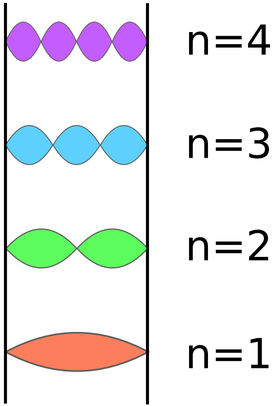
Two Plates

Two Conducting Plates



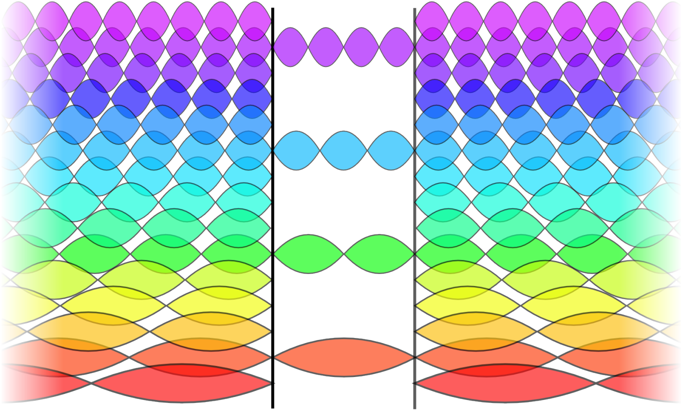
Put two metal plates very close together, with separation a.

Allowed Cavity Modes



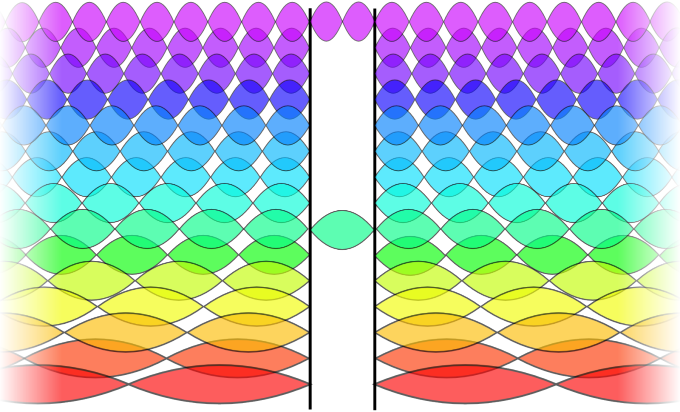
Boundary conditions ensure only certain frequencies are allowed.

External Modes



Outside the plates, all frequencies are possible.

Changing a



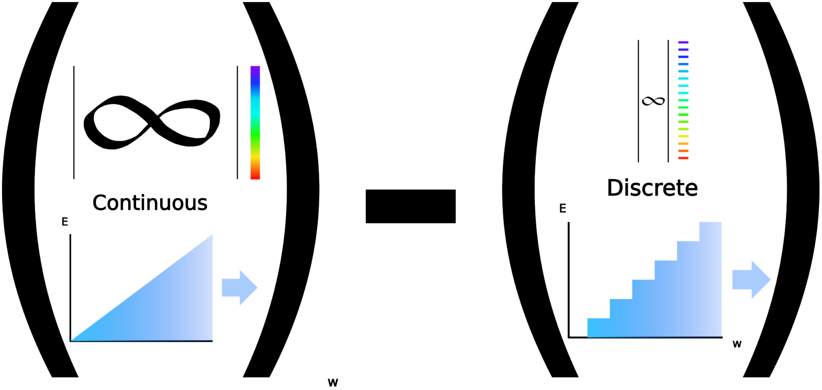
As a shrinks, modes become increasingly sparse.

Energy Difference

- As a shrinks, the density of modes in the cavity decreases
- Hence we should expect the energy within the cavity to decrease as the plates get closer together
- We'll find the energy difference by subtracting the modes in free space from the cavity modes

Difference of Infinite Quantities

What is ∞ - ∞, anyway?



Difference of Infinity Quantities

Mathematically, we're looking at the difference between a sum and an integral

$$\delta E = L^2 \hbar c \frac{\pi^2}{4a^3} \left[\sum_{n=(0)1}^{\infty} \int_0^{\infty} \sqrt{n^2+u}\, du - \int_0^{\infty} \int_0^{\infty} \sqrt{n^2+u}\, du\, dn \right]$$

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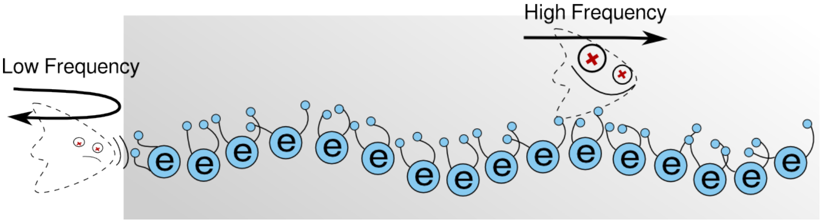
$$\delta E = L^2 \hbar c \frac{\pi^2}{4a^3} \left[\sum_{n=(0)1}^{\infty} \int_0^{\infty} \sqrt{n^2 + u}\, du - \int_0^{\infty} \int_0^{\infty} \sqrt{n^2 + u}\, du\, dn \right]$$

- ...which is still infinite.
- Great, now what?

Renormalize!



Conductors, like metals, only reflect waves up to a certain frequency.

$$\omega_p^2 = \frac{Nq^2}{m_e \epsilon_0}$$

- Boundary conditions break down above a certain frequency
- Density of modes is the same inside and out
- Neglect energy contributions above that cutoff

Regularization

We multiply the energy expression by a regulator $f(\omega)$, with the following properties:

- $f = 1$ for low frequencies
- $f = 0$ for high frequencies
- And transitions smoothly between them at some critical frequency.

Regularization: making an infinite quantity finite.

Regularized energy equation

$$\delta E = L^2 \hbar c \frac{\pi^2}{4a^3} \left[\sum_{n=(0)1}^{\infty} \int_0^{\infty} \sqrt{n^2+u}\, f(\pi\sqrt{n^2+u}/ak_m)\, du - \int_0^{\infty}\int_0^{\infty} \sqrt{n^2+u}\, f(\pi\sqrt{n^2+u}/ak_m)\, du\, dn \right]. \quad (5)$$

Renormalization

Apply Euler-Maclaurin formula

$$\int_0^N g(x)\,dx - \sum_{n=(0)1}^{(N)} g(n) = \sum_{k=2}^{p} \frac{B_k}{k!}\left(g^{(k-1)}(N) - g^{(k-1)}(0)\right) + R$$

- Gives the difference between a sum and an integral
- Depends only on value and derivatives at 0 and ∞
- Our regulator function makes this *easy*!
- Results are independent of cutoff frequency: $\infty \approx -4/720$

Renormalization: removing dependence on regularization parameters

Simplifying the renormalized energy

$$\delta E = L^2 \hbar c \frac{\pi^2}{4a^3} \left[\sum_{n=(0)1}^{\infty} \int_0^{\infty} \sqrt{n^2+u}\, f(\pi\sqrt{n^2+u}/ak_m)\, du \right.$$
$$\left. - \int_0^{\infty}\int_0^{\infty} \sqrt{n^2+u}\, f(\pi\sqrt{n^2+u}/ak_m)\, du\, dn \right].$$

Applying Euler-Maclaurin...

$$\int_0^N g(x)\, dx - \sum_{n=(0)1}^{(N)} g(n) = \sum_{k=2}^{p} \frac{B_k}{k!} \left(g^{(k-1)}(N) - g^{(k-1)}(0) \right) + R$$

We obtain the first few terms

$$\delta E = L^2 \hbar c \frac{\pi^2}{4a^3} \left[\frac{1}{12}(0-0) - \frac{1}{720}(0-(-4)) + \ldots \right]$$

Force Between Plates

The energy of the system is

$$\delta E \approx -L^2 \hbar c \frac{\pi^2}{720} \frac{1}{a^3}$$

And the pressure between the plates (force/unit area) is simply

$$\begin{aligned} P &= -\frac{\partial E}{\partial V} \\ &\approx -\hbar c \frac{\pi^2}{240} \frac{1}{a^4} \end{aligned}$$

This is the Casimir Force: pressure exerted by the quantum vacuum.

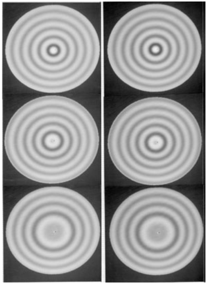
What Does This Mean?

$$F = -L^2 \hbar c \frac{\pi^2}{240} \frac{1}{a^4}$$

- Negative (pulls plates together)
- Scales linearly with area
- Depends on $\hbar$ (it's a quantum effect)
- Depends on the speed of light (changes with the medium)
- Goes as a^{-4} (falls off quickly with distance)

Geometry

Spheres



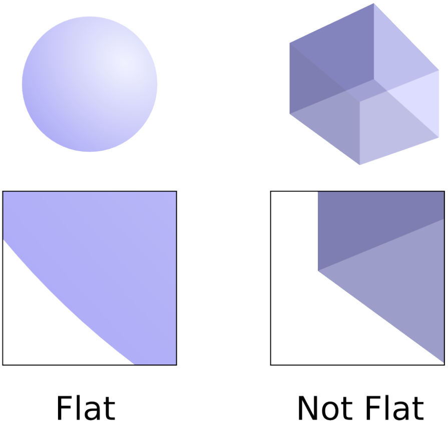
www.scielo.br

- Inside spheres, electromagnetic waves take on a different shape: Bessel functions
- Higher density of modes
- Different energy density

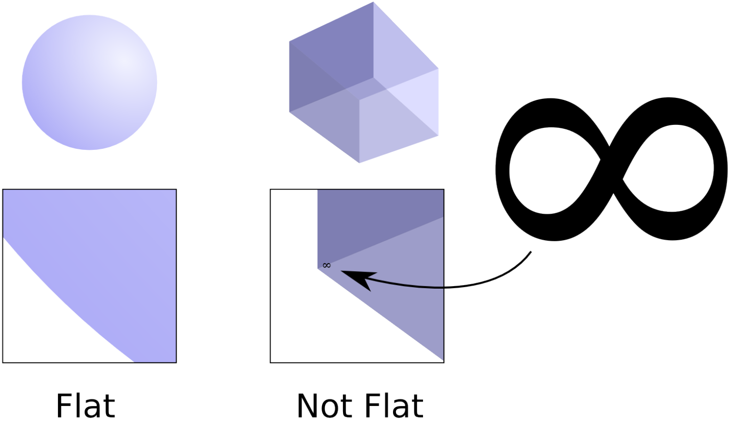
$$\langle E \rangle \approx +0.09 \frac{\hbar c}{2a}$$

- So spherical shells actually feel a *repulsive* force!

Corners and Edges



Corners and Edges

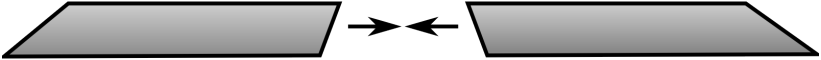


Corners and Edges

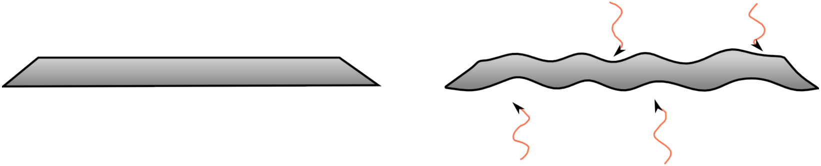
- Energy density on both sides of a smooth surface is basically the same
- Additional divergence inside corners
- Tends to flatten

Thin Foils

- A metallic sheet cut in half will try to “knit” itself back together

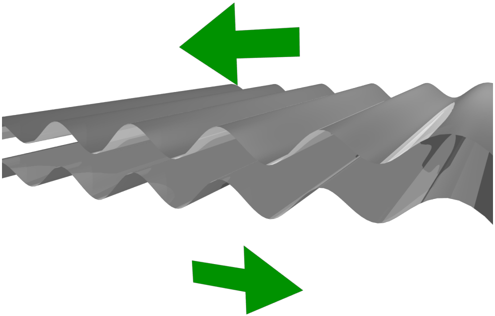


- For thin foils, the Casimir Effect magnifies normal thermal deviations
- Smooths out small ripples, creates large ones
- At nonzero temperature, foils are unstable systems!



Corrugations

- Some special configurations can even show perpendicular Casimir forces
- Two sine-wave corrugated planes
- Energetically favorable to slip sideways



Experiments

Measurement

Can we measure the Casimir effect?

- Size
- Geometry
- Roughness & Conductivity
- Thermal factors
- Electrostatic forces

Size

- We need to go *really* small to notice an effect
- For centimeter-size plates, $a \sim 10\ \mu$m
- Forces at this scale are $\sim 3.5 \times 10^{-14}$ N

Geometry

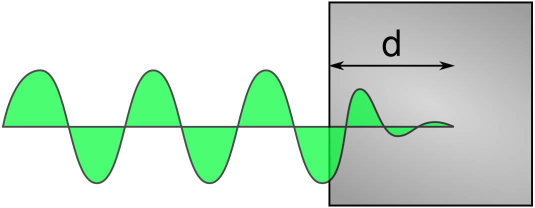
Keeping two plates parallel at 10 μm is not easy!

- Sparnay tried 2 plates in 1958: $\sim$100% margin of error
- Lamoreaux 1997: use a plane and a sphere instead
- Only one degree of freedom
- Proximity Force Theorem: basically the same as 2 plates

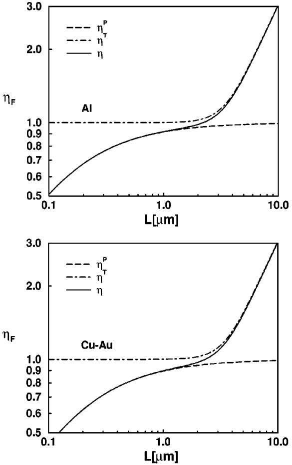
$$F(a) = 2\pi R \left(\frac{1}{3} \frac{\pi^2}{240} \frac{\hbar c}{a^3} \right)$$

Roughness & Conductivity

- No surface is perfectly flat.
- Need to treat roughness stochastically to predict corrections to vacuum energy
- Metals don't *exactly* cancel electromagnetic fields
- Waves penetrate somewhat
- At close distances, need to include corrections for finite conductivity



Thermal Corrections



- No system is at absolute zero
- Include pressure due to quantum photon gas
- Few allowed wavelengths within the cavity
- Most thermal effects due to external radiation pressure
- Corrections typically on order of 10^{-4}

Figure: Genet, et al, 2000. Force correction factors for thermal and conductivity effects, for aluminum (top) and copper/gold (bottom).

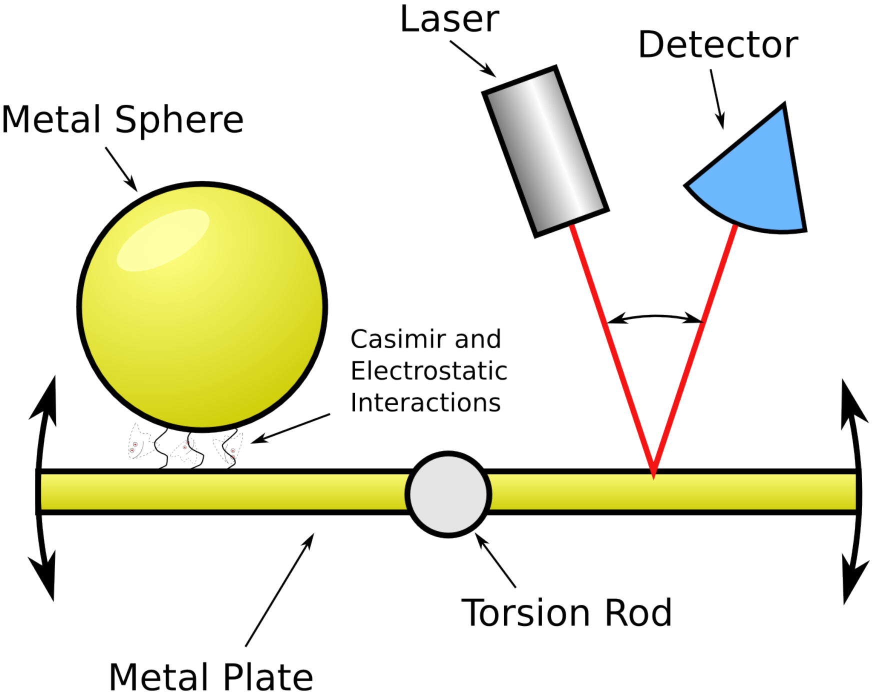
Electrostatic Forces

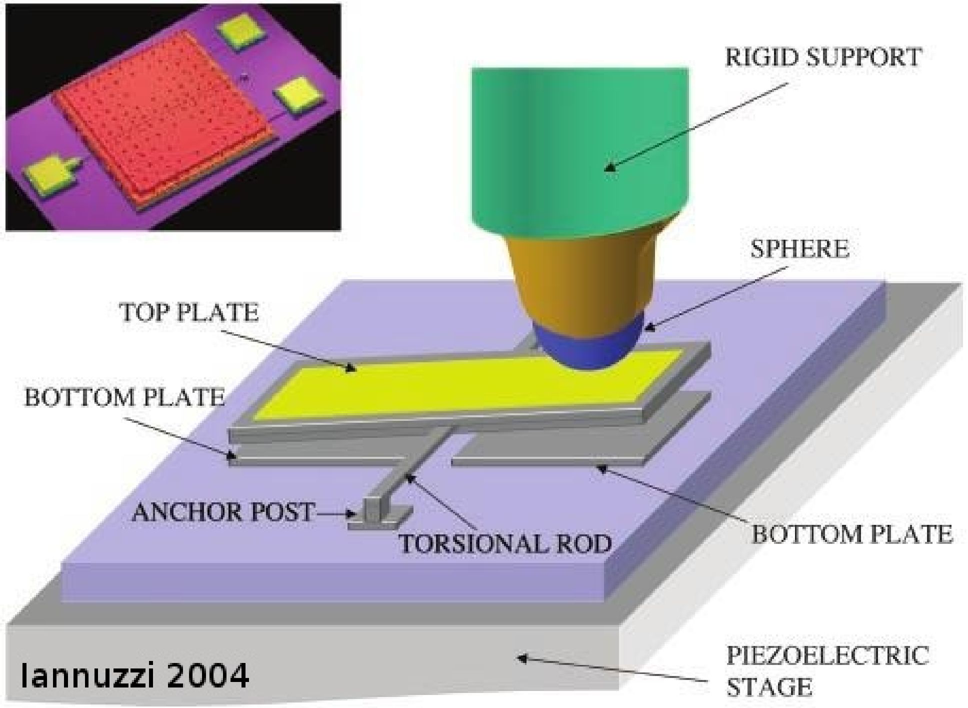
- Surfaces are metals
- Patch fields introduce additional potential
- Apply voltage to cancel most of the effect
- Extract remainder by assuming $a^{-5/4}$ dependence

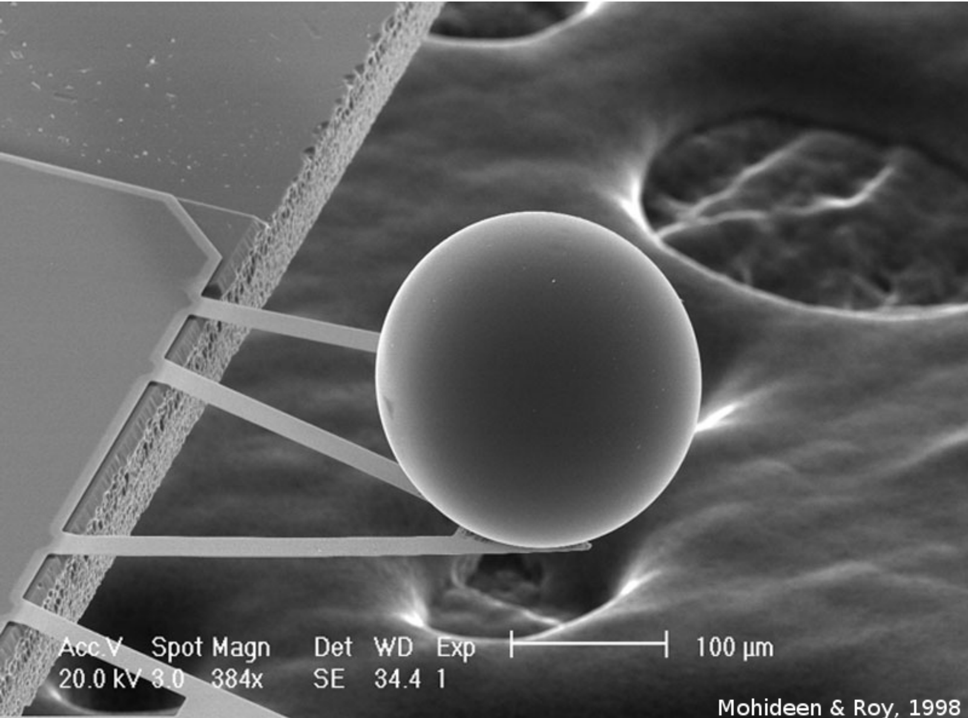
Experimental Criteria

- Fine control of distance using Piezo transducers
- Plane-sphere geometry
- Minimize roughness using thin-film deposition
- Account for dielectric behavior
- Thermal effects are small but non-negligible

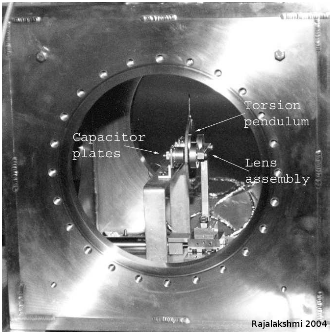
What does one of these experiments look like?

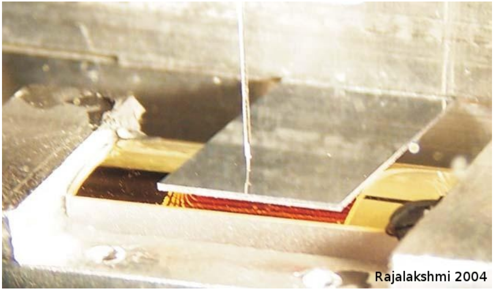

Laser
Detector
Metal Sphere
Casimir and
Electrostatic
Interactions
Torsion Rod
Metal Plate


RIGID SUPPORT
SPHERE
TOP PLATE
BOTTOM PLATE
ANCHOR POST
TORSIONAL ROD
BOTTOM PLATE
PIEZOELECTRIC
STAGE
Iannuzzi 2004


Acc.V Spot Magn Det WD Exp
20.0 kV 3.0 384x SE 34.4 1
100 µm
Mohideen & Roy, 1998

Support Structures





Timeline

1948 Casimir's plate derivation

1958 Sparnaay — parallel plate measurement (100%)

1996 Lamoreaux — torsion pendulum ball-plate (5%)

1998 Mohideen & Roy — cantilever sphere-plate (1%)

2003 Decca — dissimilar metals ($< 1\%$ close, $> 1\%$ far)

2004 Iannuzzi — hydrogen switched mirrors

Hydrogen Switched Mirrors

- Mirrors that go from reflective to transparent when you add hydrogen
- Changes what wavelengths are reflected
- . . . which results in a change in vacuum energy
- No success yet, but Iannuzzi *et al* are working on it
- We don't understand enough about the dielectric functions of the plates

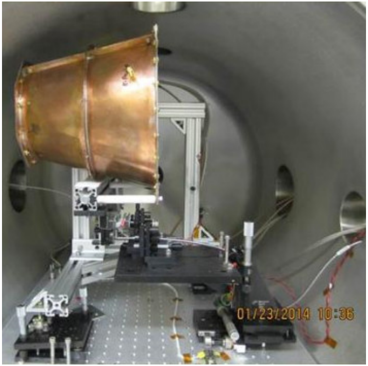
Current Directions in Research

- Compensate for contact potentials
- Better understanding of dielectric behavior
- Observe thermal corrections
- Limits of PFT

Applications

Images: memx.com, AIAA

- Nanoscale friction
- MEMs



- Magic thrusters?
- What even is conservation of 4-momentum?

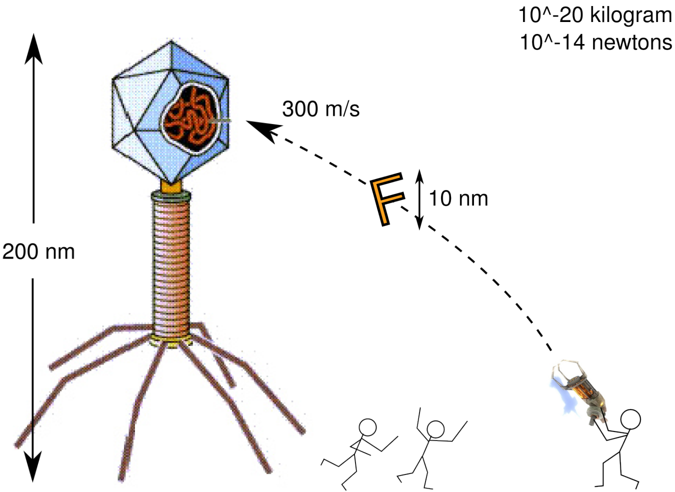
Zero-Point Energy Manipulator?

Fighting extradimensional invaders?

Zero-Point Energy Manipulator?

Fighting extradimensional invaders?
Only on small scales.

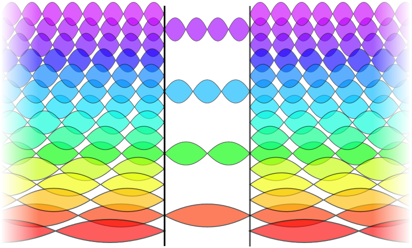
Zero-Point Energy Manipulator?



Summary

Summary

- Electromagnetic fields
- Vacuum energy
- Casimir force between plates
- Geometry dependence
- Recent experimental advances
- Nanoscale applications



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Thanks

- Hector Calderon, Faculty Advisor
- Frank McNally, Peer Advisor
- Arjendu Pattanayak & Arie Kapulkin
- Melissa Eblen-Zayas
- Cindy Blaha
- Noë Hernandez
- The 4th Olin Crew
- You!